FYJC - MATHEMATICS & STATISTICS

PAPER - I

DIFFERENTIATION

Compiled & Conducted
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DIFFERENTIATION

DERIVATIVES OF STANDARD FUNCTIONS

$\frac{d}{dx}(k) = 0$

$\frac{d}{dx}(x) = 1$

$\frac{d}{dx}(x^2) = 2x$

$\frac{d}{dx}(x^3) = 3x^2$

$\frac{d}{dx}(x^4) = 4x^3$

$\frac{d}{dx}(x^n) = nx^{n-1}$

$\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$

$\frac{d}{dx}\frac{1}{x} = \frac{-1}{x^2}$

$\frac{d}{dx}(\log x) = \frac{1}{x}$

$\frac{d}{dx}(e^x) = e^x$

$\frac{d}{dx}(a^x) = a^x.\log a$

$\frac{d}{dx}(\sin x) = \cos x$

$\frac{d}{dx}(\cos x) = -\sin x$

$\frac{d}{dx}(\tan x) = \sec 2x$

$\frac{d}{dx}(\cot x) = -\mathrm{cosec} 2x$

$\frac{d}{dx}(\sec x) = \sec x . \tan x$

$\frac{d}{dx}(\mathrm{cosec}\, x) = -\mathrm{cosec}\, x . \cot x$

Q SET -1 : SUMS ON PRODUCT RULE

01. $y = x . \tan x$

ans : $\frac{dy}{dx} = x . \sec^2 x + \tan x$

02. $y = x^2 . \tan x$

ans : $\frac{dy}{dx} = x . \sec^2 x + 2x.\tan x$

03. $y = e^x . \tan x$

ans : $\frac{dy}{dx} = e^x . (\sec^2 x + \tan x)$

04. $y = x^3 . \cos x$

ans : $\frac{dy}{dx} = 3x^2. \cos x - x^3 . \sin x$

05. $y = \sin x . \cos x$

ans : $\frac{dy}{dx} = \cos 2x$

06. $y = x^2 . 3^x$

ans : $\frac{dy}{dx} = x . 3^x . (x.\log 3 + 2)$

07. $y = x^5 . 5^x$

ans : $\frac{dy}{dx} = x^4 . 5^x . (x.\log 5 + 5)$

08. $y = (x^2 + 3x). \log x$

ans : $\frac{dy}{dx} = x + 3 + (2x + 3). \log x$

09. $y = (x^4 + 2x). \sin x$

ans : $\frac{dy}{dx} = (x^4 + 2x).\cos x + (4x^3 + 2).\sin x$

10. $y = (9x^3 - 1). \tan x$

ans : $\frac{dy}{dx} = (9x^3-1).\sec^2 x + 27x^2.\tan x$

11. $y = (4x^3 - 7x^2 + 5).\cos x$

ans : $\frac{dy}{dx} = (12x^2 - 14x).\cos x - (4x^3 - 7x^2 + 5).\sin x$

12. $y = x.\sin x + \cos x$

ans : $\frac{dy}{dx} = x.\cos x$

13. $y = 2x.\sin x - x^2.\cos x$

ans : $\frac{dy}{dx} = (x^2 + 2).\sin x$

14. $y = x^4.\log x - e^x.\sin x$

ans : $\frac{dy}{dx} = x^3 (1 + 4.\log x) - e^x (\cos x + \sin x)$

Q SET -2 : SUMS ON QUOTIENT RULE

01. $y = \frac{x^3 - 3x + 5}{2x + 1}$

ans : $\frac{dy}{dx} = \frac{4x^3 + 3x^2 - 13}{(2x + 1)^2}$

02. $y = \frac{x^3 + 2x - 1}{x^2 - 2}$

ans : $\frac{dy}{dx} = \frac{2x^4 - 8x^2 + 2x - 4}{(x^2 - 2)^2}$

03. $y = \frac{3x^2 - 4}{x + 5}$

ans : $\frac{dy}{dx} = \frac{3x^2 + 30x + 4}{(x + 5)^2}$

04. $y = \frac{(2x - 1)(3x + 1)}{4x - 1}$

ans : $\frac{dy}{dx} = \frac{24x^2 - 12x + 5}{(4x - 1)^2}$

05. $y = \frac{x + 2}{(x - 3)(x + 4)}$

ans : $\frac{dy}{dx} = \frac{-x^2 - 4x - 14}{(x^2 + x - 12)^2}$

06. $y = \frac{\sqrt{x} + 1}{\sqrt{x} - 1}$

ans : $\frac{dy}{dx} = -\frac{1}{\sqrt{x}(\sqrt{x} - 1)^2}$

07. $y = \frac{3e^x - 2}{x^2 - 4}$

ans : $\frac{dy}{dx} = \frac{e^x(3x^2 - 6x - 12) + 4x}{(x^2 - 4)^2}$

08. $y = \frac{e^x + 2}{(x - 1)(x + 5)}$

ans : $\frac{dy}{dx} = \frac{e^x.(x^2 + 2x - 9) - 4x - 8}{(x^2 + 4x - 5)^2}$

09. $y = \frac{\sin x}{1 + \cos x}$

ans : $\frac{dy}{dx} = \frac{1}{1 + \cos x}$

10. $y = \frac{x + \cos x}{1 + \sin x}$

ans : $\frac{dy}{dx} = \frac{-x.\cos x}{(1 + \sin x)^2}$

11. $y = \frac{1}{\sec x + \tan x}$

ans : $\frac{dy}{dx} = \frac{-1}{1 + \sin x}$

12. $y = \frac{2 + 3.\cos x}{3 + 2.\cos x}$

ans : $\frac{dy}{dx} = \frac{-5.\sin x}{(3 + 2.\cos x)^2}$

13. $y = \dfrac{x^3 - \sin x}{\cos x}$

ans : $\dfrac{dy}{dx} = \dfrac{3x^2.\cos x + x^3.\sin x - 1}{\cos^2 x}$

15. $y = \dfrac{1 + \tan x}{1 - \tan x}$

ans : $\dfrac{dy}{dx} = \dfrac{2}{(\cos x - \sin x)^2}$

15. $y = \dfrac{\log x}{x.e^x}$

ans : $\dfrac{dy}{dx} = \dfrac{1 - (x + 1).\log x}{x^2.e^x}$

16. $y = \dfrac{x^2 + 3}{x.\log x}$

ans : $\dfrac{dy}{dx} = \dfrac{(x^2 - 3).\log x - x^2 - 3}{(x.\log x)^2}$

17. $y = \dfrac{x^2 + 2}{x.\log x + 1}$

ans : $\dfrac{dy}{dx} = \dfrac{(x^2 - 2).\log x + 2x - x^2 - 2}{(x.\log x + 1)^2}$

18. $y = \dfrac{x.\tan x}{\sec x + \tan x}$

ans : $\dfrac{dy}{dx} = \dfrac{x . \cos x + \sin x + \sin^2 x}{(1 + \sin x)^2}$

19. $y = \dfrac{1 + \sin x}{x\sqrt{x}}$

ans : $\dfrac{dy}{dx} = \dfrac{2x. \cos x - 3 (1 + \sin x)}{2.x^2.\sqrt{x}}$

Q SET -3 : SUMS ON COMPOSITE RULE

01. $y = (2x^2 - 5)^4$

ans : $\dfrac{dy}{dx} = 16x(2x^2 - 5)^3$

02. $y = (x^2 - 3)^5$

ans : $\dfrac{dy}{dx} = 10x(x^2 - 3)^4$

03. $y = (5 - x)^3$

ans : $\dfrac{dy}{dx} = -3(5 - x)^2$

04. $y = (3x^2 - 5ax + a^2)^4$

ans : $\dfrac{dy}{dx} = 4(6x - 5a)(3x^2 - 5ax + a^2)^3$

05. $y = (x^2 - 5x + 7)^{3/2}$

ans : $\dfrac{dy}{dx} = \dfrac{3}{2}(x^2 - 5x + 7)^{1/2} . (2x - 5)$

06. $y = (3x^4 - x^3 + 4)^{5/2}$

ans : $\dfrac{dy}{dx} = \dfrac{15x^2}{2}(3x^4 - x^3 + 4)^{3/2}. (4x - 1)$

07. $y = \sqrt[8]{ax^2 + bx + c}$

ans : $\dfrac{dy}{dx} = \dfrac{1}{8}(ax^2 + bx + c)^{-7/8} . (2ax + b)$

08. $y = \sqrt[3]{(x^2 + 3x + 4/x)^2}$

ans : $\dfrac{dy}{dx} = \dfrac{2}{3}(x^2 + 3x + 4/x)^{-1/3}.(2x + 3 - 4/x^2)$

09. $y = \sqrt{1 + x^2}$

ans : $\dfrac{dy}{dx} = \dfrac{x}{\sqrt{1 + x^2}}$

10. $y = \sqrt{2x^2 + 3x - 4}$

ans : $\dfrac{dy}{dx} = \dfrac{6x^2 + 3}{2\sqrt{2x^2 + 3x - 4}}$

11. $y = \sqrt{5x^2 - 3x + 1}$

ans : $\dfrac{dy}{dx} = \dfrac{10x - 3}{2\sqrt{5x^2 - 3x + 1}}$

12. $y = \dfrac{1}{\sqrt{x^2 + 3} - \sqrt{x^2 + 2}}$

ans : $\dfrac{dy}{dx} = \dfrac{x}{\sqrt{x^2 + 3}} + \dfrac{x}{\sqrt{x^2 + 2}}$

13. $y = \dfrac{1}{\sqrt{5x^2+4} - \sqrt{5x^2+3}}$

ans : $\dfrac{dy}{dx} = \dfrac{5x}{\sqrt{5x^2+4}} + \dfrac{5x}{\sqrt{5x^2+3}}$

14. $y = \sin(3x + 4)$

ans : $\dfrac{dy}{dx} = 3.\cos(3x + 4)$

15. $y = \sec(4x - 3)$

ans : $\dfrac{dy}{dx} = 4.\sec(4x - 3).\tan(4x - 3)$

16. $y = \sec(x^2 + 1)$

ans : $\dfrac{dy}{dx} = 2x.\sec(x^2+1).\tan(x^2+1)$

17. $y = \sin(2x + 5)^2$

ans : $\dfrac{dy}{dx} = 4(2x+5).\cos(2x+5)^2$

18. $y = \tan(3x+2)^2$

ans : $\dfrac{dy}{dx} = 6(3x+2).\sec^2(3x+2)^2$

19. $y = \tan(5x-3)^2$

ans : $\dfrac{dy}{dx} = 10.(5x-3).\sec^2(5x-3)^2$

20. $y = \tan(x.e^x)$

ans : $\dfrac{dy}{dx} = e^x(x + 1).\sec^2(x.e^x)$

21. $y = \sin^5 x$

ans : $\dfrac{dy}{dx} = 5.\sin^4 x.\cos x$

22. $y = \sin^2(2x+5)$

ans : $\dfrac{dy}{dx} = 2.\sin(4x+10)$

23. $y = \sin^2(x^3)$

ans : $\dfrac{dy}{dx} = 3x^2.\sin(2x^3)$

24. $y = e^{2x^2+3}$

ans : $\dfrac{dy}{dx} = 4x.e^{2x^2+3}$

25. $y = e^{ax^2+bx+c}$

ans : $\dfrac{dy}{dx} = (2ax + b).e^{ax^2+bx+c}$

26. $y = e^{(x-1)^3}$

ans : $\dfrac{dy}{dx} = 3(x-1)^2.e^{(x-1)^3}$

27. $y = e^{x.\cos x - \sin x}$

ans : $\frac{dy}{dx} = -x . \sin x . e^{x.\cos x - \sin x}$

28 $y = \log (\sin x)$

ans : $\frac{dy}{dx} = \cot x$

29. $y = \log (\sec x + \tan x)$

ans : $\frac{dy}{dx} = \sec x$

30. $y = \log (\operatorname{cosec} x - \cot x)$

ans : $\frac{dy}{dx} = \operatorname{cosec} x$

31. $y = \log (x.\sin x + \cos x)$

ans : $\frac{dy}{dx} = \frac{x.\cos x}{x.\sin x + \cos x}$

32. $y = \log (\sin e^x)$

ans : $\frac{dy}{dx} = e^x . \cot e^x$

33. $y = \log (\tan x)$

ans : $\frac{dy}{dx} = \frac{2}{\sin 2x}$

34. $y = \log (\cos^2 5x)$

ans : $\frac{dy}{dx} = -10 . \cot 5x$

35. $y = \log (\tan 8^x)$

ans : $\frac{dy}{dx} = \frac{2.8^x \log 8}{\sin (2.8^x)}$

36. $y = 5^{(x^2+1)^3}$

ans : $\frac{dy}{dx} = 6x.(x^2+1)^2 . 5^{(x^2+1)^3} . \log 5$

37. $y = 5^{(x^2-5x+1)^2}$

$\frac{dy}{dx} = 2(x^2-5x+1) .(2x-5). 5^{(x^2-5x+1)^2} . \log 5$

38. $y = 7^{x.\sin x}$

ans : $\frac{dy}{dx} = 7^{x.\sin x} . \log 7 . (x.\cos x + \sin x)$

Q SET -4 : NM – MITHIBAI PAST PAPER SUMS

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Q SET -5 : APPLICATION OF DIFFERENTIATION

.... Pg 36

SOLUTION SET - 1

SUMS ON PRODUCT RULE

01. $y = x \,.\, \tan x$

Differentiating wrt x ;

$$\frac{dy}{dx} = x \frac{d}{dx} \tan x + \tan x \,.\, \frac{d}{dx} x$$

$$= x \,.\, \sec^2 x + \tan x \,.\, (1)$$

$$= x \,.\, \sec^2 x + \tan x$$

02. $y = x^2 \,.\, \tan x$

$$\frac{dy}{dx} = x^2 \frac{d}{dx} \tan x + \tan x \,.\, \frac{d}{dx} x^2$$

$$= x \,.\, \sec^2 x + \tan x \,.\, (2x)$$

$$= x \,.\, \sec^2 x + 2x.\tan x$$

03. $y = e^x \,.\, \tan x$

Differentiating wrt x ;

$$\frac{dy}{dx} = e^x \frac{d}{dx} \tan x + \tan x \,.\, \frac{d}{dx} e^x$$

$$= e^x \,.\, \sec^2 x + \tan x \,.\, e^x$$

$$= e^x \,.\, (\sec^2 x + \tan x)$$

04. $y = x^3 \,.\, \cos x$

$$\frac{dy}{dx} = x^3 \frac{d}{dx} \cos x + \cos x \,.\, \frac{d}{dx} x^3$$

$$= x^3 \,.\, (-\sin x) + \cos x \,.\, (3x^2)$$

$$= -x^3 \,.\, \sin x + 3x^2 .\, \cos x$$

$$= 3x^2 .\, \cos x - x^3 \,.\, \sin x$$

05. $y = \sin x \,.\, \cos x$

Differentiating wrt x ;

$$\frac{dy}{dx} = \sin x \frac{d}{dx} \cos x + \cos x \,.\, \frac{d}{dx} \sin x$$

$$= \sin x \,.\, (-\sin x) + \cos x \,.\, \cos x$$

$$= -\sin^2 x + \cos^2 x$$

$$= \cos 2x$$

06. $y = x^2 . 3^x$

Differentiating wrt x ;

$$\frac{dy}{dx} = x^2 \frac{d}{dx} 3^x + 3^x . \frac{d}{dx} x^2$$

$$= x^2 . 3^x . \log 3 + 3^x . (2x)$$

$$= 3^x . (x^2 . \log 3 + 2x)$$

$$= x . 3^x . (x.\log 3 + 2)$$

07. $y = x^5 . 5^x$

Differentiating wrt x ;

$$\frac{dy}{dx} = x^5 \frac{d}{dx} 5^x + 5^x . \frac{d}{dx} x^5$$

$$= x^5 . 5^x . \log 5 + 5^x . (5x^4)$$

$$= 5^x . (x^5 . \log 5 + 5x^4)$$

$$= x^4 . 5^x . (x.\log 5 + 5)$$

08. $y = (x^2 + 3x) . \log x$

Differentiating wrt x ;

$$\frac{dy}{dx} = (x^2 + 3x) . \frac{d}{dx} \log x + \log x \frac{d}{dx}(x^2 + 3x)$$

$$= (x^2 + 3x) . \frac{1}{x} + \log x . (2x + 3)$$

$$= \frac{x^2 + 3x}{x} + (2x + 3) . \log x$$

$$= x + 3 + (2x + 3) . \log x$$

09. $y = (x^4 + 2x) . \sin x$

Differentiating wrt x ;

$$\frac{dy}{dx} = (x^4 + 2x) . \frac{d}{dx} \sin x + \sin x \frac{d}{dx}(x^4 + 2x)$$

$$= (x^4 + 2x) . \cos x + \sin x . (4x^3 + 2)$$

$$= (x^4 + 2x) . \cos x + (4x^3 + 2) . \sin x$$

10. $y = (9x^3 - 1) . \tan x$

Differentiating wrt x ;

$$\frac{dy}{dx} = (9x^3 - 1) . \frac{d}{dx} \tan x + \tan x \frac{d}{dx}(9x^3 - 1)$$

$$= (9x^3 - 1) . \sec^2 x + \tan x . 27x^2$$

$$= (9x^3 - 1) . \sec^2 x + 27x^2 . \tan x$$

11. $y = (4x^3 - 7x^2 + 5) . \cos x$

Differentiating wrt x ;

$$\frac{dy}{dx} = (4x^3 - 7x^2 + 5) . \frac{d}{dx} \cos x + \cos x \frac{d}{dx}(4x^3 - 7x^2 + 5)$$

$$= (4x^3 - 7x^2 + 5) . (-\sin x) + \cos x . (12x^2 - 14x + 0)$$

$$= -(4x^3 - 7x^2 + 5) . \sin x + (12x^2 - 14x) . \cos x$$

$$= (12x^2 - 14x) . \cos x - (4x^3 - 7x^2 + 5) . \sin x$$

12. $y = x . \sin x + \cos x$

Differentiating wrt x ;

$$\frac{dy}{dx} = x \frac{d}{dx} \sin x + \sin x \frac{d}{dx} x + \frac{d}{dx} \cos x$$

$$= x . \cos x + \sin x . (1) - \sin x$$

$$= x . \cos x + \sin x - \sin x$$

$$= x . \cos x$$

13. $y = 2x . \sin x - x^2 . \cos x$

Differentiating wrt x ;

$$\frac{dy}{dx} = 2x \frac{d}{dx} \sin x + \sin x \frac{d}{dx} 2x - x^2 \left(\frac{d}{dx} \cos x + \cos x \frac{d}{dx} x^2 \right)$$

$$= 2x . \cos x + \sin x . 2 - \left(x^2 . (-\sin x) + \cos x . 2x \right)$$

$$= 2x . \cos x + 2 . \sin x - \left(-x^2 . \sin x + 2x . \cos x \right)$$

$$= 2x . \cos x + 2 . \sin x + x^2 . \sin x - 2x . \cos x$$

$$= 2 . \sin x + x^2 . \sin x$$

$$= (x^2 + 2) . \sin x$$

14. $y = x^4 . \log x - e^x . \sin x$

Differentiating wrt x ;

$$\frac{dy}{dx} = x^4 . \frac{d}{dx}\log x + \log x . \frac{d}{dx}x^4 - \left(e^x . \frac{d}{dx}\sin x + \sin x \frac{d}{dx}e^x\right)$$

$$= x^4 . \frac{1}{x} + \log x . 4x^3 - \left(e^x . \cos x + \sin x . e^x\right)$$

$$= x^3 + 4x^3 . \log x - e^x (\cos x + \sin x)$$

$$= x^3 (1 + 4.\log x) - e^x (\cos x + \sin x)$$

SOLUTION SET - 2

SUMS ON QUOTIENT RULE

01. $y = \dfrac{x^3 - 3x + 5}{2x + 1}$

Differentiating wrt x ;

$$\frac{dy}{dx} = \frac{(2x + 1)\ .\frac{d}{dx}(x^3 - 3x + 5) - (x^3 - 3x + 5)\frac{d}{dx}(2x + 1)}{(2x + 1)^2}$$

$$= \frac{(2x + 1)\ .(3x^2 - 3) - (x^3 - 3x + 5).\ 2}{(2x + 1)^2}$$

$$= \frac{6x^3 - 6x + 3x^2 - 3 - 2.(x^3 - 3x + 5)}{(2x + 1)^2}$$

$$= \frac{6x^3 - 6x + 3x^2 - 3 - 2x^3 + 6x - 10}{(2x + 1)^2}$$

$$= \frac{4x^3 + 3x^2 - 13}{(2x + 1)^2}$$

02. $y = \dfrac{x^3 + 2x - 1}{x^2 - 2}$

Differentiating wrt x ;

$$\frac{dy}{dx} = \frac{(x^2 - 2)\ .\frac{d}{dx}(x^3 + 2x - 1) - (x^3 + 2x - 1)\frac{d}{dx}(x^2 - 2)}{(x^2 - 2)^2}$$

$$= \frac{(x^2 - 2).(3x^2 + 2) - (x^3 + 2x - 1)\ .\ 2x}{(x^2 - 2)^2}$$

$$= \frac{3x^4 + 2x^2 - 6x^2 - 4 - 2x.(x^3 + 2x - 1)}{(x^2 - 2)^2}$$

$$= \frac{3x^4 + 2x^2 - 6x^2 - 4 - 2x^4 - 4x^2 + 2x}{(x^2 - 2)^2}$$

$$= \frac{2x^4 - 8x^2 + 2x - 4}{(x^2 - 2)^2}$$

03. $y = \dfrac{3x^2 - 4}{x + 5}$

Differentiating wrt x ;

$$\frac{dy}{dx} = \frac{(x+5)\,.\,\frac{d}{dx}(3x^2 - 4) - (3x^2 - 4)\frac{d}{dx}(x+5)}{(x+5)^2}$$

$$= \frac{(x+5)\,.\,6x - (3x^2 - 4)\,.\,1}{(x+5)^2}$$

$$= \frac{6x^2 + 30x - 3x^2 + 4}{(x+5)^2}$$

$$= \frac{3x^2 + 30x + 4}{(x+5)^2}$$

04. $y = \dfrac{(2x-1)(3x+1)}{4x-1}$

$y = \dfrac{6x^2 - x - 1}{4x - 1}$

Differentiating wrt. x ,

$$\frac{dy}{dx} = \frac{(4x-1)\,\frac{d}{dx}(6x^2 - x - 1) - (6x^2 - x - 1)\,\frac{d}{dx}(4x-1)}{(4x-1)^2}$$

$$= \frac{(4x-1)\,.\,(12x-1) - (6x^2 - x - 1)\,.\,4}{(4x-1)^2}$$

$$= \frac{48x^2 - 4x - 12x + 1 - 4(6x^2 - x - 1)}{(4x-1)^2}$$

$$= \frac{48x^2 - 16x + 1 - 24x^2 + 4x + 4}{(4x-1)^2}$$

$$= \frac{24x^2 - 12x + 5}{(4x-1)^2}$$

05. $y = \frac{x + 2}{(x - 3)(x + 4)}$

$$y = \frac{x + 2}{x^2 + x - 12}$$

Differentiating wrt x ;

$$\frac{dy}{dx} = \frac{(x^2 + x - 12)\frac{d}{dx}(x + 2) - (x + 2)\frac{d}{dx}(x^2 + x - 12)}{(x^2 + x - 12)^2}$$

$$= \frac{(x^2 + x - 12) . 1 - (x + 2)(2x + 1)}{(x^2 + x - 12\ )^2}$$

$$= \frac{x^2 + x - 12 - (2x^2 + x + 4x + 2)}{(x^2 + x - 12)^2}$$

$$= \frac{x^2 + x - 12 - (2x^2 + 5x + 2)}{(x^2 + x - 12)^2}$$

$$= \frac{x^2 + x - 12 - 2x^2 - 5x - 2}{(x^2 + x - 12)^2} = \frac{- x^2 - 4x - 14}{(x^2 + x - 12)^2}$$

06. $y = \frac{\sqrt{x} + 1}{\sqrt{x} - 1}$

Differentiating wrt x ,

$$\frac{dy}{dx} = \frac{(\sqrt{x} - 1)\frac{d}{dx}(\sqrt{x} + 1) - (\sqrt{x} + 1)\frac{d}{dx}(\sqrt{x} + 1)}{(\sqrt{x} - 1)^2}$$

$$= \frac{(\sqrt{x} - 1)\frac{1}{2\sqrt{x}} - (\sqrt{x} + 1)\frac{1}{2\sqrt{x}}}{(\sqrt{x} - 1)^2}$$

$$= \frac{\frac{1}{2\sqrt{x}}\left[\sqrt{x} - 1 - \sqrt{x} - 1\right]}{(\sqrt{x} - 1)^2}$$

$$= \frac{\frac{1}{2\sqrt{x}}(- 2)}{(\sqrt{x} - 1)^2}$$

$$= \frac{- 1}{\sqrt{x}(\sqrt{x} - 1)^2}$$

07. $y = \dfrac{3e^x - 2}{x^2 - 4}$

Differentiating wrt x ;

$$\frac{dy}{dx} = \frac{(x^2 - 4)\dfrac{d}{dx}(3e^x - 2) - (3e^x - 2)\dfrac{d}{dx}(x^2 - 4)}{(x^2 - 4)^2}$$

$$= \frac{(x^2 - 4) \,.\, 3e^x - (3e^x - 2) \,.\, 2x}{(x^2 - 4)^2}$$

$$= \frac{3e^x.(x^2 - 4) - 2x.(3e^x - 2)}{(x^2 - 4)^2}$$

$$= \frac{3e^x.x^2 - 12e^x - 6xe^x + 4x}{(x^2 - 4)^2}$$

$$= \frac{e^x(3x^2 - 6x - 12) + 4x}{(x^2 - 4)^2}$$

08. $y = \dfrac{e^x + 2}{(x - 1)(x + 5)}$

$y = \dfrac{e^x + 2}{x^2 + 4x - 5}$

Differentiating wrt x ;

$$\frac{dy}{dx} = \frac{(x^2 + 4x - 5)\dfrac{d}{dx}(e^x + 2) - (e^x + 2)\dfrac{d}{dx}(x^2 + 4x - 5)}{(x^2 + 4x - 5)^2}$$

$$= \frac{(x^2 + 4x - 5) \,.\, e^x - (e^x + 2) \,.\, (2x + 4)}{(x^2 + 4x - 5)^2}$$

$$= \frac{x^2.e^x + 4x.e^x - 5e^x - (2xe^x + 4e^x + 4x + 8)}{(x^2 + 4x - 5)^2}$$

$$= \frac{x^2.e^x + 4x.e^x - 5e^x - 2xe^x - 4e^x - 4x - 8}{(x^2 + 4x - 5)^2}$$

$$= \frac{e^x \,.\, (x^2 + 4x - 5 - 2x - 4) - 4x - 8}{(x^2 + 4x - 5)^2}$$

$$= \frac{e^x \,.\, (x^2 + 2x - 9) - 4x - 8}{(x^2 + 4x - 5)^2}$$

09. $y = \dfrac{\sin x}{1 + \cos x}$

Differentiating wrt x ;

$$\frac{dy}{dx} = \frac{(1 + \cos x)\frac{d}{dx}\sin x - \sin x \frac{d}{dx}(1 + \cos x)}{(1 + \cos x)^2}$$

$$= \frac{(1 + \cos x) . \cos x - \sin x (0 - \sin x)}{(1 + \cos x)^2}$$

$$= \frac{\cos x + \cos^2 x + \sin^2 x}{(1 + \cos x)^2}$$

$$= \frac{\cos x + 1}{(1 + \cos x)^2}$$

$$= \frac{1}{1 + \cos x}$$

10. $y = \dfrac{x + \cos x}{1 + \sin x}$

Differentiating wrt x ;

$$\frac{dy}{dx} = \frac{(1 + \sin x)\frac{d}{dx}(x + \cos x) - (x + \cos x)\frac{d}{dx}(1 + \sin x)}{(1 + \sin x)^2}$$

$$= \frac{(1 + \sin x) . (1 - \sin x) - (x + \cos x)(0 + \cos x)}{(1 + \sin x)^2}$$

$$= \frac{1 - \sin^2 x - (x + \cos x) . \cos x}{(1 + \sin x)^2}$$

$$= \frac{\cos^2 x - x.\cos x - \cos^2 x}{(1 + \sin x)^2}$$

$$= \frac{- x . \cos x}{(1 + \sin x)^2}$$

11. $y = \dfrac{1}{\sec x + \tan x}$

$y = \dfrac{\cos x}{1 + \sin x}$

Differentiating wrt x ;

$$\frac{dy}{dx} = \frac{(1 + \sin x)\dfrac{d}{dx}\cos x - \cos x\dfrac{d}{dx}(1 + \sin x)}{(1 + \sin x)^2}$$

$$= \frac{(1 + \sin x).(-\sin x) - \cos x\,(0 + \cos x)}{(1 + \sin x)^2}$$

$$= \frac{-\sin x - \sin^2 x - \cos^2 x}{(1 + \sin x)^2}$$

$$= \frac{-\sin x - (\sin^2 x + \cos^2 x)}{(1 + \sin x)^2}$$

$$= \frac{-\sin x - 1}{(1 + \sin x)^2}$$

$$= \frac{-(1 + \sin x)}{(1 + \sin x)^2} \qquad = \frac{-1}{1 + \sin x}$$

12. $y = \dfrac{2 + 3.\cos x}{3 + 2.\cos x}$

$$\frac{dy}{dx} = \frac{(3 + 2.\cos x)\dfrac{d}{dx}(2 + 3.\cos x) - (2 + 3.\cos x)\dfrac{d}{dx}(3 + 2.\cos x)}{(3 + 2.\cos x)^2}$$

$$= \frac{(3 + 2.\cos x).[0 + 3(-\sin x)] - (2 + 3.\cos x)[0 + 2(-\sin x)]}{(3 + 2.\cos x)^2}$$

$$= \frac{(3 + 2.\cos x)(-3.\sin x) - (2 + 3.\cos x).(-2.\sin x)}{(3 + 2.\cos x)^2}$$

$$= \frac{-3.\sin x\,(3 + 2.\cos x) + 2.\sin x\,(2 + 3.\cos x)}{(3 + 2.\cos x)^2}$$

$$= \frac{-9.\sin x - 6.\sin x.\cos x + 4.\sin x + 6.\sin x.\cos x}{(3 + 2.\cos x)^2}$$

$$= \frac{-5.\sin x}{(3 + 2.\cos x)^2}$$

13. $y = \dfrac{x^3 - \sin x}{\cos x}$

$$\frac{dy}{dx} = \frac{\cos x \frac{d}{dx}(x^3 - \sin x) - (x^3 - \sin x)\frac{d}{dx}(\cos x)}{\cos^2 x}$$

$$= \frac{\cos x \,.\, (3x^2 - \cos x) - (x^3 - \sin x) \,.\, (-\sin x)}{\cos^2 x}$$

$$= \frac{\cos x \,.\, (3x^2 - \cos x) + \sin x \,.\, (x^3 - \sin x)}{\cos^2 x}$$

$$= \frac{3x^2.\cos x - \cos^2 x + x^3 \,.\, \sin x - \sin^2 x}{\cos^2 x}$$

$$= \frac{3x^2.\cos x + x^3.\sin x - (\sin^2 x + \cos^2 x)}{\cos^2 x}$$

$$= \frac{3x^2.\cos x + x^3.\sin x - 1}{\cos^2 x}$$

14. $y = \dfrac{1 + \tan x}{1 - \tan x} = \dfrac{\cos x + \sin x}{\cos x - \sin x}$

$$\frac{dy}{dx} = \frac{(\cos x - \sin x)\frac{d}{dx}(\cos x + \sin x) - (\cos x + \sin x)\frac{d}{dx}(\cos x - \sin x)}{(\cos x - \sin x)^2}$$

$$= \frac{(\cos x - \sin x).(-\sin x + \cos x) - (\cos x + \sin x).(-\sin x - \cos x)}{(\cos x - \sin x)^2}$$

$$= \frac{(\cos x - \sin x).(\cos x - \sin x) + (\cos x + \sin x).(\cos x + \sin x)}{(\cos x - \sin x)^2}$$

$$= \frac{(\cos x - \sin x)^2 + (\cos x + \sin x)^2}{(\cos x - \sin x)^2}$$

$$= \frac{\cos^2 x - \cancel{2\sin x.\cos x} + \sin^2 x + \cos^2 x + \cancel{2\sin x.\cos x} + \sin^2 x}{(\cos x - \sin x)^2}$$

$$= \frac{\cos^2 x + \sin^2 x + \cos^2 x + \sin^2 x}{(\cos x - \sin x)^2}$$

$$= \frac{1 + 1}{(\cos x - \sin x)^2}$$

$$= \frac{2}{(\cos x - \sin x)^2}$$

15. $y = \dfrac{\log x}{x.e^x}$

$$\frac{dy}{dx} = \frac{x.e^x \dfrac{d}{dx}\log x - \log x \dfrac{d}{dx}(x.e^x)}{(x.e^x)^2}$$

$$= \frac{x.e^x . \dfrac{1}{x} - \log x \left(x . \dfrac{d}{dx} e^x + e^x \dfrac{d}{dx} x \right)}{(x.e^x)^2}$$

$$= \frac{e^x - \log x\ x.e^x + e^x .1}{(x.e^x)^2}$$

$$= \frac{e^x - \log x . e^x (x+1)}{(x.e^x)^2}$$

$$= \frac{e^x \left(1 - (x+1).\log x\right)}{x^2.(e^x)^2}$$

$$= \frac{1 - (x+1).\log x}{x^2.e^x}$$

16. $y = \dfrac{x^2 + 3}{x.\log x}$, Differentiating wrt x ;

$$\frac{dy}{dx} = \frac{x.\log x \dfrac{d}{dx}(x^2+3) - (x^2+3)\dfrac{d}{dx}(x.\log x)}{(x.\log x)^2}$$

$$= \frac{x.\log x . 2x - (x^2+3).\left(x \dfrac{d}{dx}\log x + \log x \dfrac{d}{dx} x \right)}{(x.\log x)^2}$$

$$= \frac{2x^2.\log x - (x^2+3)\left(x \dfrac{1}{x} + \log x .1 \right)}{(x.\log x)^2}$$

$$= \frac{2x^2.\log x - (x^2+3).(1+\log x)}{(x.\log x)^2}$$

$$= \frac{2x^2.\log x - (x^2 + x^2.\log x + 3 + 3\log x)}{(x.\log x)^2}$$

$$= \frac{2x^2.\log x - x^2 - x^2.\log x - 3 - 3\log x}{(x.\log x)^2}$$

$$= \frac{(x^2-3).\log x - x^2 - 3}{(x.\log x)^2}$$

17. $y = \dfrac{x^2 + 2}{x.\log x + 1}$

Differentiating wrt x ;

$$\frac{dy}{dx} = \frac{(x.\log x + 1)\frac{d}{dx}(x^2 + 2) - (x^2 + 2)\frac{d}{dx}(x.\log x + 1)}{(x.\log x + 1)^2}$$

$$= \frac{(x.\log x + 1) . 2x - (x^2 + 2).\left(x\frac{d}{dx}\log x + \log x\frac{d}{dx}x + 0\right)}{(x.\log x + 1)^2}$$

$$= \frac{2x^2.\log x + 2x - (x^2 + 2)\left(x\frac{1}{x} + \log x .1\right)}{(x.\log x + 1)^2}$$

$$= \frac{2x^2.\log x + 2x - (x^2 + 2).(1 + \log x)}{(x.\log x + 1)^2}$$

$$= \frac{2x^2.\log x + 2x - (x^2 + x^2.\log x + 2 + 2\log x)}{(x.\log x + 1)^2}$$

$$= \frac{2x^2.\log x + 2x - x^2 - x^2.\log x - 2 - 2\log x}{(x.\log x + 1)^2}$$

$$= \frac{(2x^2 - x^2 - 2).\log x + 2x - x^2 - 2}{(x.\log x + 1)^2}$$

$$= \frac{(x^2 - 2).\log x + 2x - x^2 - 2}{(x.\log x + 1)^2}$$

18. $y = \dfrac{x.\tan x}{\sec x + \tan x} = \dfrac{x . \sin x}{1 + \sin x}$

Differentiating wrt x ;

$$\frac{dy}{dx} = \frac{(1 + \sin x)\frac{d}{dx}(x.\sin x) - (x.\sin x)\frac{d}{dx}(1 + \sin x)}{(1 + \sin x)^2}$$

$$= \frac{(1 + \sin x)\left(x .\frac{d}{dx}\sin x + \sin x\frac{d}{dx}x\right) - (x.\sin x)(0 + \cos x)}{(1 + \sin x)^2}$$

$$= \frac{(1 + \sin x) . (x . \cos x + \sin x . 1) - x.\sin x.\cos x}{(1 + \sin x)^2}$$

$$= \frac{(1 + \sin x)(x.\cos x + \sin x) - x.\sin x.\cos x}{(1 + \sin x)^2}$$

$$= \frac{x . \cos x + \sin x + x.\sin x.\cos x + \sin^2 x - x.\sin x .\cos x}{(1 + \sin x)^2}$$

$$= \frac{x . \cos x + \sin x + \sin^2 x}{(1 + \sin x)^2}$$

19. $y = \frac{1 + \sin x}{x\sqrt{x}}$

$$\frac{dy}{dx} = \frac{x\sqrt{x}\frac{d}{dx}(1 + \sin x) - (1 + \sin x)\frac{d}{dx} x\sqrt{x}}{(x\sqrt{x})^2}$$

$$= \frac{x\sqrt{x}(0 + \cos x) - (1 + \sin x)\, x \left(\frac{d}{dx}\sqrt{x} + \sqrt{x}\frac{d}{dx} x\right)}{(x\sqrt{x})^2}$$

$$= \frac{x\sqrt{x} . \cos x - (1 + \sin x).\left(\frac{x}{2\sqrt{x}} + \sqrt{x}\right)}{(x\sqrt{x})^2}$$

$$= \frac{x\sqrt{x} . \cos x - (1 + \sin x). \frac{x + 2x}{2\sqrt{x}}}{(x\sqrt{x})^2}$$

$$= \frac{x\sqrt{x} . \cos x - (1 + \sin x). \frac{3x}{2\sqrt{x}}}{(x\sqrt{x})^2}$$

$$= \frac{x\sqrt{x} . \cos x - (1 + \sin x). \frac{3\sqrt{x}}{2}}{(x\sqrt{x})^2}$$

$$= \frac{2x\sqrt{x} . \cos x - 3\sqrt{x}\,(1 + \sin x)}{2(x\sqrt{x})^2}$$

$$= \frac{\sqrt{x}\; 2x. \cos x - 3\,(1 + \sin x)}{2.x^2(\sqrt{x})^2}$$

$$= \frac{2x. \cos x - 3\,(1 + \sin x)}{2.x^2.\sqrt{x}}$$

SOLUTION SET - 3

QSET 3 : SUMS ON COMPOSITE RULE

01. $y = (2x^2 - 5)^4$

Differentiate wrt x ;

$$\frac{dy}{dx} = 4(2x^2 - 5)3 \,.\, \frac{d}{dx}(2x^2 - 5)$$

$$= 4(2x^2 - 5)^3 \,.\, (4x - 0)$$

$$= 16x(2x^2 - 5)^3$$

02. $y = (x^2 - 3)^5$

Differentiate wrt x ;

$$\frac{dy}{dx} = 5(x^2 - 3)^4 . \frac{d}{dx}(x^2 - 3)$$

$$= 5(x^2 - 3)^4 \,.\, (2x - 0)$$

$$= 10x(x^2 - 3)^4$$

03. $y = (5 - x)^3$

Differentiate wrt x ;

$$\frac{dy}{dx} = 3(5 - x)^2 \frac{d}{dx}(5 - x)$$

$$= 3(5 - x)^2 \,.(0 - 1)$$

$$= -3(5 - x)^2$$

04. $y = (3x^2 - 5ax + a^2)^4$

Differentiate wrt x ;

$$\frac{dy}{dx} = 4(3x^2 - 5ax + a^2)^3 \frac{d}{dx}(3x^2 - 5ax + a^2)$$

$$= 4(3x^2 - 5ax + a^2)^3 . \; (6x - 5a + 0)$$

$$= 4(6x - 5a)(3x^2 - 5ax + a^2)^3$$

05. $y = (x^2 - 5x + 7)^{3/2}$

Differentiate wrt x ;

$$\frac{dy}{dx} = \frac{3}{2}(x^2 - 5x + 7)^{3/2-1} \frac{d}{dx}(x^2 - 5x + 7)$$

$$= \frac{3}{2}(x^2 - 5x + 7)^{1/2} \;.\, (2x - 5 + 0)$$

$$= \frac{3}{2}(x^2 - 5x + 7)^{1/2} \;.\, (2x - 5)$$

06. $y = (3x^4 - x^3 + 4)^{5/2}$

Differentiate wrt x ;

$$\frac{dy}{dx} = \frac{5}{2}(3x^4 - x^3 + 4)^{5/2-1} \frac{d}{dx}((3x^4 - x^3 + 4)$$

$$= \frac{5}{2}(3x^4 - x^3 + 4)^{3/2} \;.\, (12x^3 - 3x^2)$$

$$= \frac{5}{2}(3x^4 - x^3 + 4)^{3/2} \;.\, 3x^2(4x - 1)$$

$$= \frac{15x^2}{2}(3x^4 - x^3 + 4)^{3/2} \;.\, (4x - 1)$$

07. $y = \sqrt[8]{ax^2 + bx + c}$

$y = (ax^2 + bx + c)^{1/8}$

Differentiate wrt x ;

$$\frac{dy}{dx} = \frac{1}{8}(ax^2 + bx + c)^{1/8 - 1}\frac{d}{dx}(ax^2 + bx + c)$$

$$= \frac{1}{8}(ax^2 + bx + c)^{-7/8} . (2ax + b)$$

08. $y = \sqrt[3]{(x^2 + 3x + 4/x)^2}$

$y = (x^2 + 3x + 4/x)^{2/3}$

Differentiate wrt x ;

$$\frac{dy}{dx} = \frac{2}{3}(x^2 + 3x + 4/x)^{2/3-1}\frac{d}{dx}(x^2 + 3x + 4/x)$$

$$= \frac{2}{3}(x^2 + 3x + 4/x)^{-1/3} . (2x + 3 - 4/x^2)$$

09. $y = \sqrt{1 + x^2}$

Differentiate wrt x ;

$$\frac{dy}{dx} = \frac{1}{2\sqrt{1 + x^2}} . \frac{d}{dx}(1 + x^2)$$

$$= \frac{1}{2\sqrt{1 + x^2}} . 2x$$

$$= \frac{x}{\sqrt{1 + x^2}}$$

10. $y = \sqrt{2x^2 + 3x - 4}$

Differentiate wrt x ;

$$\frac{dy}{dx} = \frac{1}{2\sqrt{2x^2 + 3x - 4}}\frac{d}{dx}(2x^3 + 3x - 4)$$

$$= \frac{1}{2\sqrt{2x^2 + 3x - 4}} . (6x^2 + 3)$$

$$= \frac{6x^2 + 3}{2\sqrt{2x^2 + 3x - 4}}$$

11. $y = \sqrt{5x^2 - 3x + 1}$

` $$\frac{dy}{dx} = \frac{1}{2\sqrt{5x^2 - 3x + 1}}\frac{d}{dx}(5x^2 - 3x + 1)$$

$$= \frac{1}{2\sqrt{5x^2 - 3x + 1}} . (10x - 3)$$

$$= \frac{10x - 3}{2\sqrt{5x^2 - 3x + 1}}$$

12. $y = \dfrac{1}{\sqrt{x^2+3} - \sqrt{x^2+2}}$

$y = \dfrac{1}{\sqrt{x^2+3} - \sqrt{x^2+2}} \cdot \dfrac{\sqrt{x^2+3} + \sqrt{x^2+2}}{\sqrt{x^2+3} + \sqrt{x^2+2}}$

$y = \dfrac{\sqrt{x^2+3} + \sqrt{x^2+2}}{(x^2+3) - (x^2+2)}$

$y = \sqrt{x^2+3} + \sqrt{x^2+2}$

Differentiate wrt x ;

$\dfrac{dy}{dx} = \dfrac{1}{2\sqrt{x^2+3}} \dfrac{d}{dx}(x^2+3) + \dfrac{1}{2\sqrt{x^2+2}} \dfrac{d}{dx}(x^2+2)$

$= \dfrac{2x}{2\sqrt{x^2+3}} + \dfrac{2x}{2\sqrt{x^2+2}}$

$= \dfrac{x}{\sqrt{x^2+3}} + \dfrac{x}{\sqrt{x^2+2}}$

13. $y = \dfrac{1}{\sqrt{5x^2+4} - \sqrt{5x^2+3}}$

$= \dfrac{1}{\sqrt{5x^2+4} - \sqrt{5x^2+3}} \cdot \dfrac{\sqrt{5x^2+4} + \sqrt{5x^2+3}}{\sqrt{5x^2+4} + \sqrt{5x^2+3}}$

$= \dfrac{\sqrt{5x^2+4} + \sqrt{5x^2+3}}{(5x^2+4) - (5x^2+3)}$

$= \sqrt{5x^2+4} + \sqrt{5x^2+3}$

Differentiate wrt x ;

$\dfrac{dy}{dx} = \dfrac{1}{2\sqrt{5x^2+4}} \dfrac{d}{dx}(5x^2+4) + \dfrac{1}{2\sqrt{5x^2+3}} \dfrac{d}{dx}(5x^2+3)$

$= \dfrac{10x}{2\sqrt{5x^2+4}} + \dfrac{10x}{2\sqrt{5x^2+3}}$

$= \dfrac{5x}{\sqrt{5x^2+4}} + \dfrac{5x}{\sqrt{5x^2+3}}$

14. $y = \sin(3x+4)$

Differentiate wrt x ;

$\dfrac{dy}{dx} = \cos(3x+4) \cdot \dfrac{d}{dx}(3x+4)$

$= \cos(3x+4) \cdot 3$

$= 3.\cos(3x+4)$

15. $y = \sec(4x-3)$

Differentiate wrt x ;

$\dfrac{dy}{dx} = \sec(4x-3). \tan(4x-3). \dfrac{d}{dx}(4x-3)$

$= \sec(4x-3). \tan(4x-3) . 4$

$= 4. \sec(4x-3). \tan(4x-3)$

16. $y = \sec(x^2+1)$

Differentiate wrt x ;

$\dfrac{dy}{dx} = \sec(x^2+1) . \tan(x^2+1). \dfrac{d}{dx}(x^2+1)$

$= \sec(x^2+1) . \tan(x^2+1) . 2x$

$= 2x . \sec(x^2+1) . \tan(x^2+1)$

17. $y = \sin(2x+5)^2$

Differentiate wrt x ;

$\dfrac{dy}{dx} = \cos(2x+5)^2 \dfrac{d}{dx}(2x+5)^2$

$= \cos(2x+5)^2 . 2(2x+5) \dfrac{d}{dx}(2x+5)$

$= \cos(2x+5)^2 . 2(2x+5) . 2$

$= 4(2x+5) . \cos(2x+5)^2$

18. $y = \tan (3x+2)^2$

$$\frac{dy}{dx} = \sec^2(3x+2)^2 \frac{d}{dx}(3x+2)^2$$

$$= \sec^2(3x+2)^2 \cdot 2(3x+2) \frac{d}{dx}(3x+2)$$

$$= \sec^2(3x+2)^2 \cdot 2(3x+2) \cdot 3$$

$$= 6(3x+2) \cdot \sec^2(3x+2)^2 .$$

19. $y = \tan (5x-3)^2$

$$\frac{dy}{dx} = \sec^2(5x-3)^2 \frac{d}{dx}(5x-3)^2$$

$$= \sec^2(5x-3)^2 \cdot 2(5x-3) \frac{d}{dx}(5x-3)$$

$$= \sec^2(5x-3)^2 \cdot 2(5x-3) \cdot 5$$

$$= 10.(5x-3) \cdot \sec^2(5x-3)^2 .$$

20. $y = \tan (x.e^x)$

$$\frac{dy}{dx} = \sec^2(x.e^x) \cdot \frac{d}{dx}(x.e^x)$$

$$= \sec^2(x.e^x) \left(x \frac{d}{dx} e^x + e^x \frac{d}{dx} x \right)$$

$$= \sec^2(x.e^x) \cdot (x\, e^x + e^x .1)$$

$$= \sec^2(x.e^x) \cdot e^x(x + 1)$$

$$= e^x(x + 1) \cdot \sec^2(x.e^x)$$

21. $y = \sin^5 x$

$$\frac{dy}{dx} = 5.\sin^4 x \cdot \frac{d}{dx} \sin x$$

$$= 5. \sin^4 x \cdot \cos x$$

22. $y = \sin^2(2x+5)$

Differentiate wrt x ;

$$\frac{dy}{dx} = 2. \sin(2x+5) \cdot \frac{d}{dx} \sin (2x+5)$$

$$= 2. \sin(2x+5) .\cos (2x+5) \frac{d}{dx}(2x+5)$$

$$= 2. \sin(2x+5) \cdot \cos (2x+5) \cdot 2$$

$$= \sin (4x+10) \cdot 2$$

$$= 2 \cdot \sin (4x+10)$$

23. $y = \sin^2(x^3)$

Differentiate wrt x ;

$$\frac{dy}{dx} = 2.\sin(x^3) \cdot \frac{d}{dx} \sin(x^3)$$

$$= 2.\sin(x^3) \cdot \cos(x^3) \frac{d}{dx}(x^3)$$

$$= 2.\sin(x^3) \cdot \cos(x^3) \cdot 3x^2$$

$$= \sin (2x^3) \cdot 3x^2$$

$$= 3x^2 \cdot \sin(2x^3)$$

24.

$$y = e^{2x^2+3}$$

Differentiate wrt x ;

$$\frac{dy}{dx} = e^{2x^2+3} \cdot \frac{d}{dx}(2x^2 + 3)$$

$$= e^{2x^2+3} \cdot 4x$$

$$= 4x \cdot e^{2x^2+3}$$

25.

$$y = e^{ax^2+bx+c}$$

Differentiate wrt x ;

$$\frac{dy}{dx} = e^{ax^2+bx+c} \cdot \frac{d}{dx}(ax^2+bx+c)$$

$$= e^{ax^2+bx+c} \cdot (2ax + b)$$

$$= (2ax + b) \cdot e^{ax^2+bx+c}$$

26.

$$y = e^{(x-1)^3}$$

Differentiate wrt x ;

$$\frac{dy}{dx} = e^{(x-1)^3} \cdot \frac{d}{dx}(x-1)^3$$

$$= e^{(x-1)^3} \cdot 3(x-1)^2 \cdot \frac{d}{dx}(x - 1)$$

$$= 3(x-1)^2 \cdot e^{(x-1)^3}$$

27.

$$y = e^{x.\cos x - \sin x}$$

$$\frac{dy}{dx} = e^{x.\cos x - \sin x} \cdot \frac{d}{dx}(x.\cos x - \sin x)$$

$$= e^{x.\cos x - \sin x} \cdot \left[x \frac{d}{dx}\cos x + \cos x \frac{d}{dx} x - \frac{d}{dx}\sin x\right]$$

$$= e^{x.\cos x - \sin x} \cdot \left[x.(-\sin x) + \cos x .1 - \cos x\right]$$

$$= e^{x.\cos x - \sin x} \cdot (-x \cdot \sin x + \cos x - \cos x)$$

$$= -x \cdot \sin x \cdot e^{x.\cos x - \sin x}$$

28. $y = \log(\sin x)$

Differentiate wrt x ;

$$\frac{dy}{dx} = \frac{1}{\sin x}\frac{d}{dx}(\sin x)$$

$$= \frac{\cos x}{\sin x}$$

$$= \cot x$$

29. $y = \log(\sec x + \tan x)$

Differentiate wrt x

$$\frac{dy}{dx} = \frac{1}{\sec x + \tan x}\frac{d}{dx}(\sec x + \tan x)$$

$$= \frac{1}{\sec x + \tan x}.(\sec x.\tan x + \sec^2 x)$$

$$= \frac{\sec x(\sec x + \tan x)}{\sec x + \tan x}$$

$$= \sec x$$

30. $y = \log(\operatorname{cosec} x - \cot x)$

differentiate wrt x ;

$$\frac{dy}{dx} = \frac{1}{\operatorname{cosec} x - \cot x}\frac{d}{dx}(\operatorname{cosec} x - \cot x)$$

$$= \frac{(-\operatorname{cosec}x.\cot x + \operatorname{cosec}^2 x)}{\operatorname{cosec} x - \cot x}$$

$$= \frac{\operatorname{cosec} x(\operatorname{cosec} x - \cot x)}{\operatorname{cosec} x - \cot x}$$

$$= \operatorname{cosec} x$$

31. $y = \log(x.\sin x + \cos x)$

differentiate wrt x

$$\frac{dy}{dx} = \frac{1}{x.\sin x + \cos x}\frac{d}{dx}(x.\sin x + \cos x)$$

$$= \frac{1}{x.\sin x + \cos x}\left(x\frac{d}{dx}\sin x + \sin x\frac{d}{dx}x - \sin x\right)$$

$$= \frac{1}{x.\sin x + \cos x}(x.\cos x + \sin x - \sin x)$$

$$= \frac{x.\cos x}{x.\sin x + \cos x}$$

32. $y = \log(\sin e^x)$

Differentiate wrt x ;

$$\frac{dy}{dx} = \frac{1}{\sin e^x}\frac{d}{dx}(\sin e^x)$$

$$= \frac{1}{\sin e^x}.\cos e^x.\frac{d}{dx}e^x$$

$$= \frac{\cos e^x.e^x}{\sin e^x}$$

$$= e^x.\cot e^x$$

33. $y = \log(\tan x)$

Differentiate wrt x

$$\frac{dy}{dx} = \frac{1}{\tan x}\frac{d}{dx}\tan x$$

$$= \frac{1}{\tan x}.\sec^2 x$$

$$= \frac{1}{\frac{\sin x}{\cos x}}.\frac{1}{\cos^2 x}$$

$$= \frac{2}{2\sin x.\cos x} = \frac{2}{\sin 2x}$$

34. $y = \log(\cos^2 5x)$

$y = 2.\log(\cos 5x)$

Differentiate wrt x ;

$$\frac{dy}{dx} = 2\,\frac{1}{\cos 5x}\,\frac{d}{dx}\cos 5x$$

$$= 2\,\frac{1}{\cos 5x}(-\sin 5x)\quad\frac{d}{dx}5x$$

$$= 2\,\frac{1}{\cos 5x}(-\sin 5x)\;.\;5$$

$$= -10\;.\;\cot 5x$$

35. $y = \log(\tan 8^x)$

Differentiate wrt x ;

$$\frac{dy}{dx} = \frac{1}{\tan 8^x}\;\frac{d}{dx}\;(\tan 8^x)$$

$$= \frac{1}{\tan 8^x}\;.\;\sec^2 8^x\;.\;\frac{d}{dx}8^x$$

$$= \frac{1}{\tan 8^x}.\sec^2 8^x\;.\;8^x\;.\;\log 8$$

$$= \frac{1}{\frac{\sin 8^x}{\cos 8^x}}\;.\;\frac{1}{\cos^2 8^x}\;.\;8^x.\log 8$$

$$= \frac{2}{2.\sin 8^x.\cos 8^x}.\;8^x.\log 8$$

$$= \frac{2.8^x.\log 8}{\sin(2.8^x)}$$

36. $y = 5^{(x^2+1)^3}$

Differentiate wrt x ;

$$\frac{dy}{dx} = 5^{(x^2+1)^3}\;.\log 5\;.\;\frac{d}{dx}\;(x^2+1)^3$$

$$= 5^{(x^2+1)^3}\;.\log 5\;.\;3(x^2+1)^2.\frac{d}{dx}\;(x^2+1)$$

$$= 5^{(x^2+1)^3}\;.\log 5\;.\;3(x^2+1)^2.\;\;2x$$

$$= 6x.(x^2+1)^2\;.\;5^{(x^2+1)^3}\;.\log 5$$

37. $y = 5^{(x^2-5x+1)^2}$

Differentiate wrt x ;

$$\frac{dy}{dx} = 5^{(x^2-5x+1)^2}\;.\log 5\;.\frac{d}{dx}\;(x^2-5x+1)^2$$

$$= 5^{(x^2-5x+1)^2}\;.\log 5.\;2(x^2-5x+1)\;.\frac{d}{dx}(x^2-5x+1)$$

$$= 5^{(x^2-5x+1)^2}\;.\log 5\;.\;2(x^2-5x+1)\;.(2x-5)$$

$$= 2(x^2-5x+1)\;.(2x-5)\;.\;5^{(x^2-5x+1)^2}\;.\log 5$$

38. $y = 7^{x.\sin x}$

$$\frac{dy}{dx} = 7^{x.\sin x} . \log 7 . \frac{d}{dx} x.\sin x$$

$$= 7^{x.\sin x} . \log 7 . \left(x \frac{d}{dx} \sin x + \sin x \frac{d}{dx} x \right)$$

$$= 7^{x.\sin x} . \log 7 . (x.\cos x + \sin x)$$

01. $y = (2x^3 - 7)^5 . \log(\tan x)$

STEP 1 :

$\frac{d}{dx}\log(\tan x)$

$= \frac{1}{\tan x} . \frac{d}{dx}\tan x$

$= \frac{1}{\tan x} . \sec^2 x$

$= \frac{1}{\frac{\sin x}{\cos x}} . \frac{1}{\cos^2 x}$

$= \frac{1}{\sin x . \cos x}$

$= \frac{2}{2\sin x . \cos x}$

$= \frac{2}{\sin 2x} \quad = 2 . \text{cosec } 2x$

STEP 2 : $\frac{d}{dx}(2x^3 - 7)^5$

$= 5(2x^3 - 7)^4 . \frac{d}{dx}(2x^3 - 7)$

$= 5(2x^3 - 7)^4 . 6x^2$

$= 30x^2.(2x^3 - 7)^4$

STEP 3 :

$y = (2x^3 - 7)^5 . \log(\tan x)$

$\frac{dy}{dx} = (2x^3 - 7)^5 \frac{d}{dx}\log(\tan x) + \log(\tan x) \frac{d}{dx}(2x^3 - 7)^5$

$= (2x^3-7)^5 . 2\text{cosec}2x + \log(\tan x).30x^2.(2x^3 - 7)^4$

$= (2x^3-7)^5 . 2\text{cosec}2x + 30x^2.(2x^3 - 7)^4.\log(\tan x)$

$= (2x^3-7)^4 \left[2(2x^3-7)\text{cosec}2x + 30x^2..\log(\tan x) \right]$

02. $y = \dfrac{\sin(4x^2 - 3)}{(x^2 - 2)^4}$

STEP 1 :

$\dfrac{d}{dx} \sin(4x^2 - 3)$

$= \cos(4x^2 - 3) \,.\, \dfrac{d}{dx}(4x^2 - 3)$

$= \cos(4x^2 - 3) \,.\, 8x$

$= 8x \,.\, \cos(4x^2 - 3)$

STEP 2

$\dfrac{d}{dx}(x^2 - 2)^4$

$= 4(x^2 - 2)^3 .\ \dfrac{d}{dx}(x^2 - 2)$

$= 4(x^2 - 2)^3 \,.\, 2x \qquad = 8x\,(x^2 - 2)^3$

STEP 3

$y = \dfrac{\sin(4x^2 - 3)}{(x^2 - 2)^4}$

$\dfrac{dy}{dx} = \dfrac{(x^2-2)^4 \dfrac{d}{dx}\sin(4x^2-3) - \sin(4x^2-3)\dfrac{d}{dx}(x^2-4)^3}{\left[(x^2 - 2)^4\right]^2}$

$= \dfrac{(x^2-4)^4 .8x \,.\, \cos(4x^2-3) - \sin(4x^2-3)\ .8x(x^2 - 2)^3}{(x^2 - 2)^8}$

ARRANGING THE TERMS

$= \dfrac{8x.\ (x^2-2)^4 .\ \cos(4x^2-3) - .\ 8x(x^2 - 2)^3 \sin(4x^2-3)}{(x^2 - 2)^8}$

$= \dfrac{8x(x^2 - 2)^3 \left[(x^2-2).\ \cos(4x^2-3) - \sin(4x^2-3)\right]}{(x^2 - 2)^8}$

$= \dfrac{8x \left[(x^2-2).\ \cos(4x^2-3) - \sin(4x^2-3)\right]}{(x^2 - 2)^5}$

03. $y = \dfrac{x.\cos^2 x}{(1 + x)^3}$

STEP 1 :

$$\frac{d}{dx}\, x.\cos^2 x$$

$$= x \frac{d}{dx}\cos^2 x + \cos^2 x . \frac{d}{dx} x$$

$$= x.2 \cos x \frac{d}{dx}\cos x + \cos^2 x . 1$$

$$= x. 2 \cos x (-\sin x) + \cos^2 x$$

$$= -x . 2 \sin x \cos x + \cos^2 x$$

$$= \cos^2 x - x.\sin 2x$$

STEP 2 :

$$\frac{d}{dx}(1 + x)^3$$

$$= 3(1 + x)^2 \frac{d}{dx}(1 + x)$$

$$= 3(1 + x)^2$$

STEP 3 :

$$y = \frac{x.\cos^2 x}{(1 + x)^3}$$

$$\frac{dy}{dx} = \frac{(1+x)^3 \frac{d}{dx} x.\cos^2 x - x.\cos^2 x \frac{d}{dx}(1+x)^3}{\left[(1 + x)^3\right]^2}$$

$$= \frac{(1+x)^3 (\cos^2 x - x.\sin 2x) - x.\cos^2 x . 3(1+x)^2}{(1 + x)^6}$$

ARRANGING THE TERMS

$$= \frac{(1+x)^3 (\cos^2 x - x.\sin 2x) - 3(1+x)^2.x.\cos^2 x}{(1 + x)^6}$$

$$= \frac{(1+x)^2 \left[(1+x) (\cos^2 x - x.\sin 2x) - 3x.\cos^2 x\right]}{(1 + x)^6}$$

$$= \frac{(1+x) (\cos^2 x - x.\sin 2x) - 3x.\cos^2 x}{(1 + x)^4}$$

04. $y = \dfrac{\log(\cos 5x)}{x^2 + 3x - 1}$

STEP 1 :

$$\frac{d}{dx}\log(\cos 5x)$$

$$= \frac{1}{\cos 5x} \; \frac{d}{dx}\cos 5x$$

$$= \frac{1}{\cos 5x} . (-\sin 5x) . \frac{d}{dx}5x$$

$$= \frac{1}{\cos 5x} . (-\sin 5x) . 5$$

$$= -5 . \tan 5x$$

STEP 2 :

$$y = \frac{\log(\cos 5x)}{x^2 + 3x - 1}$$

$dy/dx =$

$$\frac{(x^2+3x-1) \; \frac{d}{dx}\log(\cos 5x) - \log(\cos 5x) \; \frac{d}{dx}(x^2+3x-1)}{(x^2+3x-1)^2}$$

$$= \frac{(x^2+3x-1) \quad (-5.\tan 5x) - \log(\cos 5x) . (2x+3)}{(x^2+3x-1)^2}$$

$$= \frac{-5 \; (x^2+3x-1) \quad .\tan 5x \; - \; (2x+3). \log(\cos 5x)}{(x^2+3x-1)^2}$$

05. $y = \dfrac{x^4 + 4^x}{8 + \sin x}$

$$\frac{dy}{dx} = \frac{(8 + \sin x)\frac{d}{dx}(x^4 + 4^x) - (x^4 + 4^x)\frac{d}{dx}(8 + \sin x)}{(8 + \sin x)^2}$$

$$= \frac{(8 + \sin x)(4x^3 + 4^x.\log 4) \quad + (x^4 + 4^x).\cos x}{(8 + \sin x)^2}$$

06. $y = \log(\sin e^x) + \sqrt{5 + x^6} \cdot \sec x$

STEP 1 :

$$\frac{d}{dx}\log(\sin e^x)$$

$$= \frac{1}{\sin e^x}\,\frac{d}{dx}\sin e^x$$

$$= \frac{1}{\sin e^x} \cdot \cos e^x \cdot \frac{d}{dx}\, e^x$$

$$= \frac{1}{\sin e^x} \cdot \cos e^x \cdot e^x$$

$$= e^x \cdot \cot e^x$$

STEP 2 :

$$\frac{d}{dx}\sqrt{5 + x^6} \cdot \sec x$$

$$= \sqrt{5 + x^6} \cdot \frac{d}{dx}\sec x + \sec x\, \frac{d}{dx}\sqrt{5 + x^6}$$

$$= \sqrt{5+x^6} \cdot \sec x.\tan x + \sec x \,\frac{1}{2\sqrt{5+x^6}}\,\frac{d}{dx}(5+x^6)$$

$$= \sqrt{5+x^6} \cdot \sec x.\tan x + \sec x \,\frac{1}{2\sqrt{5+x^6}}\,6x^5$$

$$= \sqrt{5+x^6} \cdot \sec x.\tan x + \sec x \,\frac{3x^5}{\sqrt{5+x^6}}$$

$$= \sec x \left(\sqrt{5+x^6} \cdot \tan x + \frac{3x^5}{\sqrt{5+x^6}}\right)$$

STEP 3 :

$$y = \log(\sin e^x) + \sqrt{5 + x^6} \cdot \sec x$$

$$\frac{dy}{dx} = e^x.\cot e^x + \sec x \left(\sqrt{5+x^6} \cdot \tan x + \frac{3x^5}{\sqrt{5+x^6}}\right)$$

07. $y = \dfrac{\sec^3 x}{e^{4x}.(1+x)^5}$

STEP 1 :

$$\frac{d}{dx}\sec^3 x = 3\sec^2 x . \frac{d}{dx}\sec x$$

$$= 3\sec^2 x . \sec x . \tan x$$

$$= 3\sec^3 x.\tan x$$

STEP 2 :

$$\frac{d}{dx} e^{4x}.(1+x)^5$$

$$= e^{4x}. \frac{d}{dx}(1+x)^5 + (1+x)^5 . \frac{d}{dx} e^{4x}$$

$$= e^{4x}. 5(1+x)^4 \frac{d}{dx}(1+x) + (1+x)^5 . e^{4x} \frac{d}{dx} 4x$$

$$= e^{4x}. 5(1+x)^4 + (1+x)^5 . e^{4x} . 4$$

$$= 5.e^{4x}. (1+x)^4 + 4.e^{4x}. (1+x)^5$$

$$= e^{4x}. (1+x)^4 \left(5 + 4.(1+x)\right)$$

$$= e^{4x}. (1+x)^4 (9 + 4x)$$

STEP 3 :

$$\frac{dy}{dx} = \frac{e^{4x}.(1+x)^5 \frac{d}{dx} \sec^3 x - \sec^3 x \frac{d}{dx} e^{4x}.(1+x)^5}{\left(e^{4x}.(1+x)^5\right)^2}$$

$$= \frac{e^{4x}.(1+x)^5 \; 3\sec^3 x .\tan x - \sec^3 x . e^{4x}. (1+x)^4 (9+4x)}{\left(e^{4x}.(1+x)^5\right)^2}$$

$$= \frac{3e^{4x}.(1+x)^5 \sec^3 x \tan x - e^{4x}.(1+x)^4 (9+4x).\sec^3 x}{\left(e^{4x}.\right)^2 (1+x)^{10}}$$

$$= \frac{e^{4x}.(1+x)^4 \sec^3 x \left(3(1+x)\tan x - (9+4x)\right)}{\left(e^{4x}\right)^2 (1+x)^{10}}$$

$$= \frac{\sec^3 x \left(3(1+x)\tan x - (9+4x)\right)}{e^{4x}. (1+x)^6}$$

08. $y = \sin^3 3x . e^{\sqrt{x}} + \log \dfrac{x+1}{\sqrt{x^2+1}}$

STEP 1

$$\frac{d}{dx} \sin^3 3x . e^{\sqrt{x}}$$

$$= \sin^3 3x . \frac{d}{dx} e^{\sqrt{x}} + e^{\sqrt{x}} \frac{d}{dx} \sin^3 3x$$

$$= \sin^3 3x . e^{\sqrt{x}} \frac{d}{dx} \sqrt{x} + e^{\sqrt{x}} 3 \sin^2 3x \frac{d}{dx} \sin 3x$$

$$= \sin^3 3x . e^{\sqrt{x}} \frac{1}{2\sqrt{x}} + e^{\sqrt{x}} 3\sin^2 3x . \cos 3x \frac{d}{dx} 3x$$

$$= \sin^3 3x . e^{\sqrt{x}} \frac{1}{2\sqrt{x}} + e^{\sqrt{x}} 3 \sin^2 3x . \cos 3x \ 3$$

$$= \frac{e^{\sqrt{x}} . \sin^3 3x .}{2\sqrt{x}} + 9 e^{\sqrt{x}} \sin^2 3x . \cos 3x$$

$$= e^{\sqrt{x}} . \sin^2 3x \left(\frac{\sin 3x}{2\sqrt{x}} + 9 . \cos 3x \right)$$

STEP 2 :

$$\frac{d}{dx} \log \frac{x+1}{\sqrt{x^2+1}}$$

$$= \frac{d}{dx} \left[\log (x+1) - \log \sqrt{x^2+1} \right]$$

$$= \frac{d}{dx} \left[\log (x+1) \frac{-1}{2} \log (x^2+1) \right]$$

$$= \frac{1}{x+1} \frac{d}{dx} (x+1) - \frac{1}{2} \frac{1}{x^2+1} \frac{d}{dx} (x^2+1)$$

$$= \frac{1}{x+1} - \frac{1}{2} \frac{1}{x^2+1} 2x$$

$$= \frac{1}{x+1} - \frac{x}{x^2+1}$$

STEP 3 :

$$\frac{dy}{dx} = e^{\sqrt{x}} . \sin^2 3x \left(\frac{\sin 3x}{2\sqrt{x}} + 9\cos 3x \right) + \frac{1}{x+1} - \frac{x}{x^2+1}$$

09. $y = \dfrac{\sin\sqrt{x^2+2} + \log(xe^x)}{5^{x\tan x}}$.

HINTS $\dfrac{d}{dx}\sin\sqrt{x^2+2} = \dfrac{x.\cos\sqrt{x^2+2}}{\sqrt{x^2+2}}$

$$\frac{d}{dx}\log(xe^x) = \frac{x+1}{x}$$

$$\frac{d}{dx}\left[\sin\sqrt{x^2+2} + \log(xe^x)\right] = \frac{x.\cos\sqrt{x^2+2}}{\sqrt{x^2+2}} + \frac{x+1}{x}$$

$$\frac{d}{dx}5^{x\tan x}. = 5^{x\tan x}.\log 5 . (x\sec^2 x + \tan x)$$

now

$$\frac{dy}{dx} = \frac{5^{x\tan x}\dfrac{d}{dx}\left[\sin\sqrt{x^2+2} + \log(xe^x)\right] - \left[\sin\sqrt{x^2+2} + \log(xe^x)\right]\dfrac{d}{dx}5^{x\tan x}.}{(5^{x\tan x})^2}$$

$$= \frac{5^{x\tan x}\left(\dfrac{x.\cos\sqrt{x^2+2}}{\sqrt{x^2+2}} + \dfrac{x+1}{x}\right) - \left[\sin\sqrt{x^2+2} + \log(xe^x)\right]5^{x\tan x}.\log 5 .(x\sec^2 x + \tan x)}{(5^{x\tan x})^2}$$

$$= \frac{5^{x\tan x}\left(\dfrac{x.\cos\sqrt{x^2+2}}{\sqrt{x^2+2}} + \dfrac{x+1}{x} - \left[\sin\sqrt{x^2+2} + \log(xe^x)\right]\log 5 .(x\sec^2 x + \tan x)\right)}{(5^{x\tan x})^2}$$

$$= \frac{\left(\dfrac{x.\cos\sqrt{x^2+2}}{\sqrt{x^2+2}} + \dfrac{x+1}{x} - \left[\sin\sqrt{x^2+2} + \log(xe^x)\right]\log 5 .(x\sec^2 x + \tan x)\right)}{5^{x\tan x}}$$

QUESTION SET

01. the relation between price (P) and demand (D) of a cup of tea is given as

$$D = \frac{32}{P}$$

Find the rate at which the demand changes when the price is Rs 4/- . Interpret the result

ans : –2

02. the demand D for a price P is given as

$$D = \frac{27}{P}$$

Find the rate of change of demand when price is 3

ans : –3

03. the demand D of biscuits at price P is given as $D = \frac{64}{P^3}$

Find the marginal demand , when price is Rs 2/–

ans : –12

04. for a commodity , price – demand relationship is given as

$$D = \frac{P + 5}{P - 1}$$

Find marginal demand when price is 2

ans : –6

05. the demand function of a commodity is given as $P = 20 + D - D^2$. Find the rate at which price is changing when demand is 3

ans : –5

06. the relation between supply S and price P of a good is given as $S = 2P^2$. Find the marginal supply at price Rs 5/–. Interpret the result

ans : 20

07. The supply S of electric bulbs at price P is given by $S = 2P^3 + 7$

Find the marginal supply when price is 5/– . Interpret the result

ans : 150

08. the supply S for a commodity at price P is given by $S = P^2 + 9P - 2$

Find the marginal supply when price is 7

ans : 23

09. For a commodity ,

demand $D = \frac{24P}{P - 2}$ and

Suppy $S = P^2$

Find equilibrium price . Find marginal demand and marginal supply at that price

ans : 6 , –3 , 12

10. the demand function of a commodity is given by $P = 32 + 3D - D^2$

Find the rate at which the price is changing , when demand is 2 . Also find the rate at which the total Revenue R is changing at that time

ans : –1 , 32

11. the demand function is given as

$$P = 175 + 9D + 25D^2$$

Find the total revenue and marginal revenue when demand is 10

ans : 27650 , 7855

12. The price P and the demand x of pens has relation $x = 20\sqrt{P} - 4$

Find the marginal revenue when P = 9

ans : 86

13. the cost C for an output x is given as

$$C = x^4 - 2x^3 + 80x + 150$$

Find the rate at which the cost is changing when output x is 2

ans : 88

14. The total cost of producing x items is given by $C = x^2 + 4x + 4$

Find the average cost and the marginal cost .

What is the marginal cost when x = 7

ans : $x + 4 + 4/x$, $2x + 4$, 18

15. the cost of producing x – articles is given by

$$C = x^2 + 15x + 81$$

Find the average cost and marginal cost functions . Find the marginal cost when x = 10 . Find x for which the marginal cost equals average cost

ans : x+15+81/x , 2x+15 , 35 , 9

16. if the total cost function is given by ;

$$C = 5x^3 + 2x^2 + 7$$

Find the average cost and the marginal cost when x = 4

ans : 359/4 , 256

17. the total cost of x pencils is given by

$$C = 15 + 28x - x^2$$

Find x when the marginal cost is 20 . Find the average cost at this value of x

ans : 4 , 111/4

18. the total cost of producing n note books is given by

$$C = 1500 - 75n + 2n^2 + \frac{n^3}{5}$$

Find the marginal cost at n = 10

ans : 25

19. the total cost of 't' toy cars is given by

$$C = 5(2^t) + 17$$

Find the marginal cost and average cost at t = 3

ans : 40.log2 , 19

SOLUTION SET

01. the relation between price (P) and demand (D) of a cup of tea is given as

$$D = \frac{32}{P}$$

Find the rate at which the demand changes when the price is Rs 4/- . Interpret the result

SOLUTION

$D = \frac{32}{P}$

Rate of change of demand when P = 4

$= \frac{dD}{dP}$

$= \frac{-32}{P^2}$

Put p = 4

$= \frac{-32}{16}$

$= -2$, Demand falls at p = 4

02. the demand D for a price P is given as

$$D = \frac{27}{P}$$

Find the rate of change of demand when price is 3

SOLUTION

$D = \frac{27}{P}$

Rate of change of demand when P = 4

$= \frac{dD}{dP}$

$= \frac{-27}{P^2}$

Put p = 3

$= \frac{-27}{9}$

$= -3$

03. the demand D of biscuits at price P is given as $D = \frac{64}{P^3}$

Find the marginal demand , when price is Rs 2/–

SOLUTION

$D = \frac{64}{P^3} = 64P^{-3}$

Marginal demand at P = 2

$= \frac{dD}{dP}$

$= 64(-3P^{-4})$

$= \frac{-192}{P^4}$

Put P = 2

$= \frac{-192}{16}$

$= -12$

04. for a commodity , price – demand relationship is given as

$$D = \frac{P+5}{P-1}$$

Find marginal demand when price is 2

SOLUTION

$D = \frac{P+5}{P-1}$

Marginal Demand when p = 2

$= \frac{dD}{dP}$

$$= \frac{(P-1)\frac{d}{dP}(P+5) - (P+5)\frac{d}{dP}(P-1)}{(P-1)^2}$$

$= \frac{(P-1).1 - (P+5).1}{(P-1)^2}$

$= \frac{p-1-p-5}{(P-1)^2}$

$= \frac{-6}{(P-1)^2}$.

Put p = 2

$= -6$

05. the demand function of a commodity is given as $P = 20 + D - D^2$. Find the rate at which price is changing when demand is 3

SOLUTION :

$P = 20 + D - D^2$

Rate of change of price when D = 3

$= \frac{dP}{dD}$

$= 1 - 2D$

Put D = 3

$= 1 - 6$

$= -5$

06. the relation between supply S and price P of a good is given as $S = 2P^2$. Find the marginal supply at price Rs 5/–. Interpret the result

SOLUTION :

$S = 2P^2$

Marginal supply at P = 5

$= \frac{dS}{dP}$

$= 4P$

Put P = 5

$= 20$; supply increases with price

07. The supply S of electric bulbs at price P is given by $S = 2P^3 + 7$
Find the marginal supply when price is 5/– . Interpret the result

SOLUTION :

$S = 2P^3 + 7$

Marginal supply at P = 5

$= \frac{dS}{dP}$

$= 6P^2$

Put P = 5

$= 6(25)$

$= 150$; supply increases with price

08. the supply S for a commodity at price P is given by $S = P^2 + 9P - 2$
Find the marginal supply when price is 7

SOLUTION :

$S = P^2 + 9P - 2$

Marginal supply at P = 5

$= \frac{dS}{dP}$

$= 2P + 9$

Put P = 7

$= 14 + 9$

$= 23$

09. For a commodity , demand $D = \frac{24P}{P - 2}$ and

Suppy $S = P^2$
Find equilibrium price . Find marginal demand and marginal supply at that price

SOLUTION :

EQUILIBRIUM PRICE

$D = S$

$\frac{24P}{P - 2} = P^2$

$\frac{24}{P-2} = P$

$24 = P(P - 2)$

$6(4) = P(P - 2) \quad \therefore P = 6$

$D = \frac{24P}{P-2}$

Marginal Demand at P = 6

$= \frac{dD}{dP}$

$= \frac{(P-2)\frac{d}{dP}24P - 24P\frac{d}{dP}(P-2)}{(P-2)^2}$

$= \frac{(P-2).24 - 24P(1)}{(P-2)^2}$

$= \frac{24P - 48 - 24P}{(P-2)^2}$

$= \frac{-48}{(P-2)^2}$

Put P = 6

$= \frac{-48}{(6-2)^2}$

$= \frac{-48}{16} \quad = -3$

$S = P^2$

Marginal Supply at P = 6

$= \frac{dS}{dP}$

$= 2P$

put P = 6

$= 12$

10. the demand function of a commodity is given by $P = 32 + 3D - D^2$
Find the rate at which the price is changing , when demand is 2 . Also find the rate at which the total Revenue R is changing at that time

SOLUTION

$P = 32 + 3D - D^2$

Rate of change of price when D = 2

$= \frac{dP}{dD}$

$= 3 - 2D$

Put D = 2

$= 3 - 4$

$= -1$

TOTAL REVENUE

$R = pD$

$= (32 + 3D - D^2).D$

$= 32D + 3D^2 - D^3$

Rate of change of total Revenue R when D = 2

$= \frac{dR}{dD}$

$= 32 + 6D - 3D^2$

Put D = 2

$= 32 + 12 - 3(4)$

$= 32 + 12 - 12$

$= 32$

11. the demand function is given as
$P = 175 + 9D + 25D^2$
Find the total revenue and marginal revenue when demand is 10

SOLUTION

Total Revenue

$R = pD$

$= (175 + 9D + 25D^2).D$

$= 175D + 9D^2 + 25D^3$

Put D = 10

$= 1750 + 900 + 25000$

$= 27650$

Marginal Revenue when D = 10

$= \frac{dR}{dD}$

$= 175 + 18D + 75D^2$

Put D = 10

$= 175 + 180 + 7500$

$= 7855$

12. The price P and the demand x of pens has relation $x = 20\sqrt{P} - 4$
Find the marginal revenue when P = 9

SOLUTION :

Total Revenue

$R = p.x$

$= P(20\sqrt{P} - 4)$

$= 20P\sqrt{P} - 4P$

$= 20P^{3/2} - 4P$

Marginal Revenue when P = 9

$= \frac{dR}{dP}$

$= 20 . \frac{3}{2} P^{1/2} - 4$

$= 30\sqrt{P} - 4$

Put P = 9

$= 30(3) - 4$

$= 86$

13. the cost C for an output x is given as
$C = x^4 - 2x^3 + 80x + 150$
Find the rate at which the cost is changing when output x is 2

SOLUTION

Rate of change of cost when x = 2

$= \frac{dC}{dx}$

$= 4x^3 - 6x^2 + 80$

Put $x = 2$

$= 4(8) - 6(4) + 80$

$= 32 - 24 + 80$

$= 88$

14. The total cost of producing x items is given by $C = x^2 + 4x + 4$
Find the average cost and the marginal cost .
What is the marginal cost when x = 7

SOLUTION :

$C = x^2 + 4x + 4$

Average Cost

$= \frac{C}{x}$

$= \frac{x^2 + 4x + 4}{x}$

$= x + 4 + \frac{4}{x}$

Marginal cost when x = 7

$= \frac{dC}{dx}$

$= 2x + 4$

Put $x = 7$

$= 14 + 4$

$= 18$

15. the cost of producing x – articles is given by

$$C = x^2 + 15x + 81$$

Find the average cost and marginal cost functions . Find the marginal cost when x = 10 . Find x for which the marginal cost equals average cost

SOLUTION :

$C = x^2 + 15x + 81$

Average Cost

$= \frac{C}{x}$

$= \frac{x^2 + 15x + 81}{x}$

$= x + 15 + \frac{81}{x}$

Marginal cost when x = 10

$= \frac{dC}{dx}$

$= 2x + 15$

Put $x = 10$

$= 20 + 15$

$= 35$

For what value of x ,
Marginal cost = average cost

$2x + 15 = x + 15 + \frac{81}{x}$

$x = \frac{81}{x}$

$x^2 = 81$

$x = 9$

(output x cannot be negative)

16. if the total cost function is given by ;

$$C = 5x^3 + 2x^2 + 7$$

Find the average cost and the marginal cost when x = 4

SOLUTION :

$C = 5x^3 + 2x^2 + 7$

Average Cost when x = 4

$= \frac{C}{x}$

$= \frac{5x^3 + 2x^2 + 7}{x}$

$= 5x^2 + 2x + \frac{7}{x}$

Put $x = 4$

$= 5(16) + 2(4) + \frac{7}{4}$

$= 80 + 8 + \frac{7}{4}$

$= 88 + \frac{7}{4}$

$= \frac{352 + 7}{4}$

$= 359/4$

Marginal cost when x = 4

$= \frac{dC}{dx}$

$= 15x^2 + 4x$

Put x = 4

$= 15(16) + 4(4)$

$= 240 + 16$

$= 256$

17. the total cost of x pencils is given by

$C = 15 + 28x - x^2$

Find x when the marginal cost is 20 . Find the average cost at this value of x

SOLUTION :

Marginal cost = 20

$\frac{dC}{dx} = 20$

$28 - 2x = 20$

$8 = 2x$

$x = 4$

Average cost at x = 4

$= \frac{C}{x}$

$= \frac{15 + 28x - x^2}{x}$

$= \frac{15}{x} + 28 - x$

Put x = 4

$= \frac{15}{4} + 28 - 4$

$= \frac{15}{4} + 24$

$= \frac{15 + 96}{4}$

$= \frac{111}{4}$

18. the total cost of producing n note books is given by

$C = 1500 - 75n + 2n^2 + \frac{n^3}{5}$

Find the marginal cost at n = 10

SOLUTION :

Marginal cost at n = 10

$= \frac{dC}{dn}$

$= -75 + 4n + \frac{3n^2}{5}$

Put n = 10

$= -75 + 40 + \frac{3(100)}{5}$

$= -75 + 40 + 60 = 25$

19. the total cost of 't' toy cars is given by

$C = 5(2^t) + 17$

Find the marginal cost and average cost at t = 3

SOLUTION :

Marginal Cost at t = 3

$= \frac{dC}{dt}$

$= 5(2^t.\log 2)$

Put t = 3

$= 5(2^3.\log 2)$

$= 5(8.\log 2)$

$= 40\log 2$

Average cost at t = 3

$= \frac{C}{t}$

$= \frac{5(2^t) + 17}{t}$

Put t = 3

$= \frac{5(2^3) + 17}{3}$

$= \frac{40 + 17}{3}$

$= \frac{57}{3}$

$= 19$